NASA TECHNICAL MEMORANDUM

NASA TM X-1943

NASA TM X-1943

INFERENCE OF TEMPERATURE AND WATER-VAPOR STRUCTURE IN THE STRATOSPHERE FROM LIMB RADIANCE PROFILES

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1970

1. Report No. NASA TM X-1943	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle INFERENCE OF TEMPERATURE AND WATER-VAPOR STRUCTURE IN THE STRATOSPHERE FROM LIMB RADIANCE PROFILES		5. Report Date February 1970
		6. Performing Organization Code
7. Author(s) Thomas B. McKee		8. Performing Organization Report No. L-7050
		10. Work Unit No. 715-02-00-01-23
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23365		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Memorandum
		14. Sponsoring Agency Code

15. Supplementary Notes

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17. Key Words Suggested by Author(s)	18. Distribution Statement
Stratospheric temperature Stratospheric water-vapor structure	Unclassified – Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	23	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

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SUMMARY

Techniques to infer stratospheric temperature and water-vapor mixing ratio as a function of pressure from limb radiance profiles have been presented. Requirements for accurate pointing information have been eliminated, and only the easily obtained vertical scan rate is needed. Example calculations of inferred temperatures and water-vapor mixing ratios from unperturbed and perturbed radiance profiles are given. Radiance profiles for the 615 cm^{-1} to 715 cm^{-1} spectral region due to thermal emission of carbon dioxide were used for temperature inference, and radiance profiles for the 205 cm^{-1} to 295 cm^{-1} spectral region due to thermal emission of water vapor were used for mixing-ratio inference. Pressure ranges of the inferred values are 0.3 to 100 millibars (30 to 10 000 N/m^2) for temperature and 3 to 200 millibars for water-vapor mixing ratio. Errors in inferred quantities due to radiance errors have been examined and are not excessive.

INTRODUCTION

Techniques to infer atmospheric temperature and water-vapor structure in the stratosphere from limb (horizon) radiance profiles have been developed and demonstrated. References 1 and 2 discuss inference of temperature, illustrate the different approaches taken to obtain temperature structure from limb radiance profiles, and present examples based on measured data. References 2 and 3 discuss techniques to infer water-vapor mixing ratio and present examples based on measured radiance profiles.

All of the techniques previously reported require radiance as a function of tangent height in order to infer temperature or water-vapor mixing ratio as a function of geometric altitude. The need to measure tangent height accurately places a severe restraint on any system of measuring limb radiance profiles. A pointing accuracy of 1 km at the limb from an altitude of 500 km corresponds to an angular accuracy of 0.023 degree, which is difficult to achieve. Inferring atmospheric characteristics from limb radiance profiles would clearly be a much simpler task if the need for accurate pointing were eliminated.

The purpose of this report is to derive an inference technique which does not require accurate pointing data. A technique of inferring temperature and water-vapor mixing ratio as functions of pressure from limb radiance profiles without knowledge of tangent height is presented.

SYMBOLS

C_1	constant, 1.1909×10^{-5} erg-centimeter^2/second-steradian
C_2	constant, 1.4389 centimeters-degrees Kelvin
g	acceleration of gravity, kilometers/second^2
H	tangent height, kilometers
h	altitude, kilometers
J_ν	source function, watts/meter^2-steradian-centimeter^{-1}
N	radiance, watts/meter^2-steradian
N_c	calculated radiance, watts/meter^2-steradian
N_m	measured radiance, watts/meter^2-steradian
N_ν	spectral radiance, watts/meter^2-steradian-centimeter^{-1}
P	pressure at tangent point, millibars
p	pressure, millibars (1 millibar = 10^2 newtons/meter^2)
R	gas constant for air, 2.87×10^6 erg/gram-degree Kelvin
r	earth radius, kilometers
s	distance along line of sight, kilometers
T	temperature, degrees Kelvin

T_{av}	average temperature for layer, degrees Kelvin
t	time, seconds
γ	lapse rate, degrees Kelvin/kilometer
θ	nadir angle, degrees
ν	wave number, centimeter^{-1}
ν_l	lower wave-number limit of integration, centimeter^{-1}
ν_u	upper wave-number limit of integration, centimeter^{-1}
σ	standard deviation
τ	transmittance

Subscripts:

a	value at center of wave band
i	integer
o	value at upper boundary of iteration technique
1,2,3,4	points along line of sight

DERIVATION OF TECHNIQUE

The technique employed to infer atmospheric temperature and constituent mixing-ratio structure is an iterative process applied to the equation of radiative transfer used to calculate limb radiance profiles. A discussion of the calculation of radiance profiles and the derivation of the technique for inference are presented.

Calculation of Radiance Profiles

An instrument (radiometer) with a small field of view is positioned outside the atmosphere to measure limb radiance profiles as it scans across the limb. One line of sight through the atmosphere is shown in figure 1. The radiance exiting the atmosphere

along a line of sight is given by the equation of radiative transfer (see ref. 4) as

$$N = -\int_{\nu} \int_{s} J_{\nu} \frac{\partial \tau}{\partial s} \, ds \, d\nu + \int_{\nu} J_{\nu} \tau_1 \, d\nu \tag{1}$$

An evaluation of equation (1) along different lines of sight yields a radiance profile. In reference 1 each line of sight is defined by its tangent height H. However, the point where the line of sight forms a right angle with the radius is also uniquely defined by the total atmospheric pressure P at that point. For the present discussion, a limb radiance profile is defined as the radiance from different lines of sight as a function of pressure at the tangent point.

Two restrictions applied to equation (1) are that no boundaries (earth or clouds) are encountered along the line of sight and that only thermal radiation from atmospheric gases contributes to the source function J_{ν}. Then equation (1) becomes

$$N(P) = -\int_{\nu} \int_{s} N_{\nu} \frac{\partial \tau}{\partial s} \, ds \, d\nu \tag{2}$$

where

$$N_{\nu} = \frac{C_1 \nu^3}{\exp\left(\frac{C_2 \nu}{T}\right) - 1}$$

The meteorological variables – temperature, pressure, and mixing ratio – are needed to evaluate equation (2). The spectral radiance N_{ν} is a function of temperature only. The transmittance is weakly dependent on temperature but strongly dependent on pressure and the amount of absorbing gas; thus, transmittance is primarily influenced by mixing ratio and pressure.

The integration of equation (2) over the variables can be transformed to an integration of that equation over pressure p. From figure 1 the distance from the tangent point outward is given by

$$s^2 = (r + h)^2 - (r + H)^2 = 2r(h - H) + h^2 - H^2 \tag{3}$$

Differentiation of equation (3) with r and H considered constants for one line of sight yields

$$s \, ds = r \, dh + h \, dh \tag{4a}$$

An approximation is possible here since $r >> h$. The range of h involved is where thermal emission by carbon dioxide and water vapor is significant. Because very little radiation originates above 70 km, h is only 1 percent or less of r; thus, the second term of equation 4(a) can be omitted to yield

$$s \, ds = r \, dh \tag{4b}$$

If the atmosphere is considered to be an ideal gas in hydrostatic equilibrium, the hydrostatic equation is

$$dp = -\frac{pg}{RT} \, dh \tag{5}$$

Combining equation (4b) and equation (5) yields

$$s \, ds = -\frac{rRT}{pg} \, dp \tag{6}$$

which is integrated to

$$s^2 = -2rR \int_P^p \frac{T \, dp}{pg} \tag{7}$$

Equations (6) and (7) are used with equation (2) to compute limb radiance profiles without using altitudes. A radiance profile computed for the 615-cm^{-1} to 715-cm^{-1} spectral region due to thermal emission of carbon dioxide is shown in figure 2. Computations were made by using the 1962 U.S. Standard Atmosphere (ref. 5) and a carbon dioxide mixing ratio of 314 parts per million. Equation (2) was evaluated, as described in reference 6, with transmittance data from reference 7. Doppler broadening was accounted for. In figure 2 the values computed by using pressure are compared with those computed by using altitude (ref. 1).

A radiance profile computed by using pressure for the 205-cm^{-1} to 295-cm^{-1} spectral band due to thermal emission of water vapor is shown in figure 3. Computations were made by using the 1962 U.S. Standard Atmosphere with the mixing ratios shown in figure 3. Transmittance data were obtained from reference 8.

Inference of Temperature

Inference of temperature requires three assumptions:

(1) Transmittance model

(2) Mixing ratio as a function of pressure

(3) Absence of clouds

Examples given in the present discussion are for the 615-cm^{-1} to 715-cm^{-1} spectral region of the carbon dioxide emission, but any suitable gas and spectral region could be used.

The iterative technique to obtain temperature begins at small radiance values and proceeds to larger values. At the first level to be inferred, the radiance is calculated as

$$N_c(P_o) = -\int_{\nu_l}^{\nu_u} \int_1^{\tau_1} N_\nu(T)\, d\tau\, d\nu \tag{8}$$

where the atmosphere above P_o is assumed to be hydrostatic and have a constant lapse rate with height such that T at any p is given by

$$T = T_o\left(\frac{p}{P_o}\right)^{\frac{-R\gamma}{g}} \tag{9}$$

Initial values of T_o, P_o, and γ are assumed based on correlation of these parameters with radiances calculated from the climatological atmospheres in reference 9. From a measured radiance profile, the radiance error between calculated and measured values is

$$\Delta N(P_o) = N_m(P_o) - N_c(P_o) \tag{10}$$

At this point a ΔT, which is a function of $\Delta N(P_o)$, is calculated, as in reference 1, by

$$\Delta T = \frac{\left[N_m(P_o) - N_c(P_o)\right]T_o^2}{N_{\nu,a}(T_o)C_2\nu_a \int_{\nu_l}^{\nu_u} \int_1^{\tau_1} d\tau\, d\nu} \tag{11}$$

where the limits $\tau = 1$ to τ_1 account for the transmittance through the atmosphere. When T_o is changed by this ΔT, the new values of T above the P_o level are

computed from equation (9). The values of P_o and γ remain unchanged. A new value of $N_c(P_o)$ is computed, and this procedure is repeated until $\Delta N(P_o)$ is made small.

The calculation proceeds to the next larger radiance value. To calculate a radiance value for comparison with the measured radiance, the location of the line of sight in the atmosphere must be known. An examination of the radiance measurement, the geometry, and the hydrostatic condition enables specification of the pressure at the tangent point of the line of sight without accurate pointing information. A limb radiance profile is generated as an instrument scans across the limb as a function of time. The geometry yields a relationship between tangent height and nadir angle as

$$r + H = (r + h) \sin \theta$$

For a limb scan, the radius is constant and observer altitude is constant so that the time derivative is

$$\frac{dH}{dt} = (r + h) \cos \theta \frac{d\theta}{dt} \tag{12}$$

The value of $r + h$ is determined from tracking data, and $\cos \theta$ can be predicted to an accuracy of 2 percent or better by knowledge of the spectral region of the measurement for observer altitudes of 500 km or higher. Then the vertical scan rate $d\theta/dt$, which is an easily obtained parameter, must be measured for the experiment in order that dH/dt can be determined. Now, the hydrostatic equation can be expressed in terms of the variables which define the tangent point as

$$dP = -\frac{pg}{RT} dH$$

For small increments of dP and dH, the upper boundary is P_{i-1}, the lower is P_i, and the average is $\frac{P_i + P_{i-1}}{2}$. Then from the hydrostatic equation

$$P_i = P_{i-1} \frac{2RT_{av} + g\Delta H}{2RT_{av} - g\Delta H}$$

is obtained. The change in tangent height is ΔH and is expressed by $(dH/dt)\Delta t$ which leads to

$$P_i = P_{i-1} \frac{2RT_{av} + g(dH/dt)\Delta t}{2RT_{av} - g(dH/dt)\Delta t} \tag{13}$$

Equation (13) provides the specification of the pressure of the tangent point needed to calculate a radiance and insures that the layer is in hydrostatic balance. The ΔH used must be kept small so that equation (13), still a differential form of the hydrostatic equation, is valid.

Equation (13) is used with $P_{i-1} = P_o$ and $T_{av} = T_o$ to estimate the new pressure P_i. A radiance is next computed for the ith layer and the radiance error is expressed as

$$\Delta N_i(P_i) = N_{m,i}(P_i) - N_{c,i}(P_i) \tag{14}$$

From ΔN_i, a ΔT_i is calculated, as in reference 1, to be

$$\Delta T_i = \frac{\left[N_m(P_i) - N_c(P_i)\right] T_i^2}{N_{\nu,a}(T_i) C_2 \nu_a \int_{\nu_l}^{\nu_u} \int_{\tau_3}^{\tau_2} d\tau \, d\nu} \tag{15}$$

where the limits τ_2 and τ_3 are used; these limits account only for the transmittance of the new layer. Equations (14), (15), and (13) are used in an iterative operation until ΔN_i is small and the new layer is in hydrostatic balance. The calculation then proceeds to the next radiance value and, thus, works down through the atmosphere layer by layer.

Inference of Water-Vapor Mixing Ratio

Inference of water-vapor mixing ratio requires four assumptions:

(1) Transmittance model

(2) Temperature as a function of pressure

(3) Central pressure of line of sight

(4) Absence of clouds

Assumptions (2) and (3) are met by optically alining the radiometer used for measuring water-vapor emission with the radiometer used to gather data for temperature inference. Water-vapor mixing ratio is assumed constant above the highest level for which radiance data are available. A radiance is calculated from equation (2) and a radiance error formed for the first measured radiance as

$$\Delta N(P_o) = N_m(P_o) - N_c(P_o)$$

The mixing ratio is changed and $\Delta N(P_o)$ determined until $\Delta N(P_o)$ is made small.

The next radiance point is selected and a radiance calculated for the new line of sight. Equation (2) is expanded to give

$$N(P_i) = -\int_\nu \int_{\tau_4}^{\tau_3} N_\nu \, d\tau \, d\nu - \int_\nu \int_{\tau_3}^{\tau_2} N_\nu \, d\tau \, d\nu - \int_\nu \int_{\tau_2}^{\tau_1} N_\nu \, d\tau \, d\nu \tag{16}$$

The integral from τ_3 to τ_2 gives a contribution from the new layer which was not present in the line of sight designated by P_o. Mixing ratio in the segment τ_3 to τ_2 is the only unknown needed to evaluate equation (16). A mixing ratio equal to the one in the layer immediately above is initially chosen to compute an $N_c(P_i)$ which is used to form the radiance error of

$$\Delta N(P_i) = N_m(P_i) - N_c(P_i)$$

Mixing ratio is changed until $\Delta N(P_i)$ is made small, and the inference moves downward layer by layer. The method to infer water-vapor mixing ratio is the same in principle as that reported in reference 3.

DISCUSSION

The technique derived herein is ideally suited to a satellite experiment for mapping temperature and water vapor in the stratosphere over the entire earth. Spectral regions desirable for a satellite experiment are not necessarily the ones used here for examples. In an actual experiment, two spectral regions could be chosen to measure carbon dioxide emission and, thus, extend the results to higher and lower pressures. A method of determining the initial pressure could be developed with two intervals much better than with one interval; thus, no extensive work has been done involving only one interval.

The technique to infer water-vapor structure is applicable to any constituent of the atmosphere which has a measurable thermal emission and for which transmittance can be calculated. This technique is readily applicable to ozone (O_3) which can be determined simultaneously with water vapor.

EXAMPLES OF INFERENCE

Figure 4 illustrates the inference of temperature from an unperturbed radiance profile calculated by using the 1962 U.S. Standard Atmosphere (fig. 2) and temperatures inferred from the same radiance profile perturbed with scale, bias, and random radiance errors. Pressure ranges of the inferred values are 0.3 to 100 millibars for temperature.

Errors used are not unreasonable with present radiometric technology. In the unperturbed inference, the temperature error at the smallest pressure is caused by assuming an incorrect lapse rate for the atmosphere above the first level for which temperature was inferred. A lapse rate of 4° K/km was used when the lapse rate of the standard atmosphere is changing from 2° K/km to 4° K/km. Perturbed examples do not show excessive sensitivity to radiance errors. The diameter of the symbol used in figure 4 is about 2° K. Figure 4(e) illustrates the effect of an error in the assumed initial pressure P_0. The error in P_0 of 10 percent produces larger temperature errors than the radiance errors shown and indicates that care must be taken in developing a method to start the inference.

Inference of water-vapor mixing ratio is shown in figure 5 for an unperturbed radiance profile and for scale, bias, and random radiance errors. Pressure ranges of the inferred values are 3 to 200 millibars for water-vapor mixing ratio. Errors introduced are the same as those used in the temperature examples. Scale error produces the largest errors in inferred mixing ratio, and these errors become greater at larger pressures.

CONCLUDING REMARKS

Techniques to infer stratospheric temperature and water-vapor mixing ratio as a function of pressure from limb radiance profiles have been presented. Requirements for accurate pointing information have been eliminated, and only the easily obtained vertical scan rate is needed. Example calculations of inferred temperatures and water-vapor mixing ratio from unperturbed and perturbed radiance profiles are given. Radiance profiles for the 615 cm^{-1} to 715 cm^{-1} spectral region due to thermal emission of carbon dioxide were used for temperature inference, and radiance profiles for the 205 cm^{-1} to 295 cm^{-1} spectral region due to thermal emission of water vapor were used for mixing-ratio inference. Effects of scale, bias, and random radiance errors are shown. Pressure ranges of the inferred values are 0.3 to 100 millibars (30 to 10 000 N/m^2) for temperature and 3 to 200 millibars for water-vapor mixing ratio. Errors in inferred quantities due to radiance errors are not excessive.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., November 25, 1969.

REFERENCES

1. McKee, Thomas B.; Whitman, Ruth I.; and Lambiotte, Jules J., Jr.: A Technique to Infer Atmospheric Temperature From Horizon Radiance Profiles. NASA TN D-5068, 1969.

2. House, Frederick B.; and Ohring, George: Inference of Stratospheric Temperature and Moisture Profiles From Observations of the Infrared Horizon. NASA CR-1419, 1969.

3. McKee, Thomas B.; Whitman, Ruth I.; and Lambiotte, Jules J., Jr.: A Technique to Infer Atmospheric Water-Vapor Mixing Ratio From Measured Horizon Radiance Profiles. NASA TN D-5252, 1969.

4. Goody, R. M.: Atmospheric Radiation I. - Theoretical Basis. Clarendon Press (Oxford), 1964.

5. Anon.: U.S. Standard Atmosphere, 1962. NASA, U.S. Air Force, and U.S. Weather Bur., Dec. 1962.

6. Bates, Jerry C.; Hanson, David S.; House, Fred B.; Carpenter, Robert O'B.; and Gille, John C.: The Synthesis of 15μ Infrared Horizon Radiance Profiles From Meteorological Data Inputs. NASA CR-724, 1967.

7. Stull, V. R.; Wyatt, P. J.; and Plass, G. N.: Infrared Transmission Studies. Final Report, Volume III - The Infrared Absorption of Carbon Dioxide. SSD-TDR-62-127, Vol. III, U.S. Air Force, Jan. 31, 1963.

8. Elsasser, Walter M.; and Culbertson, Margaret F.: Atmospheric Radiation Tables. Meteorol. Monogr., vol. 4, no. 23, Aug. 1960.

9. Anon.: U.S. Standard Atmosphere Supplements, 1966. Environ. Sci. Serv. Admin., NASA, and U.S. Air Force.

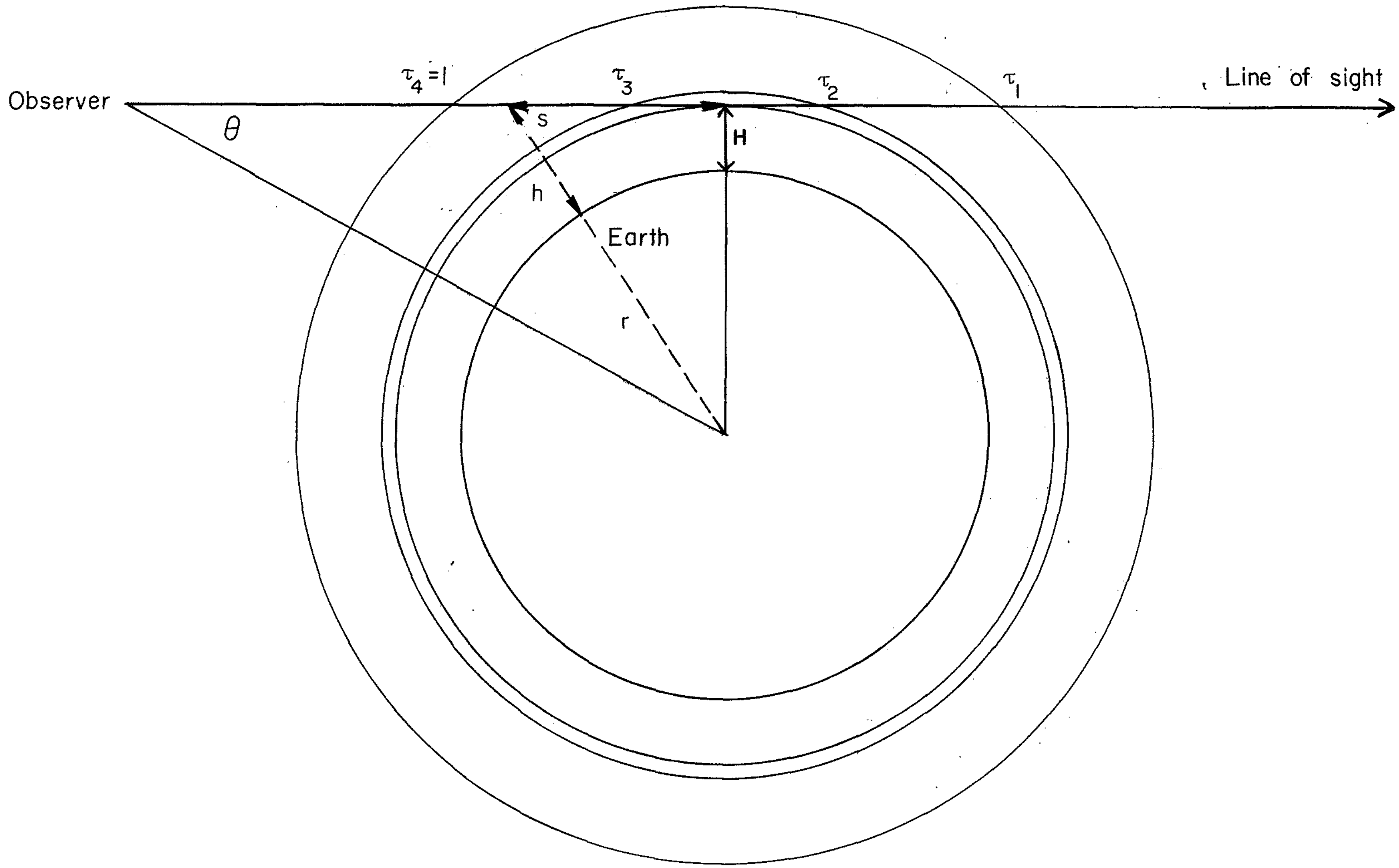


Figure 1.- Limb geometry.

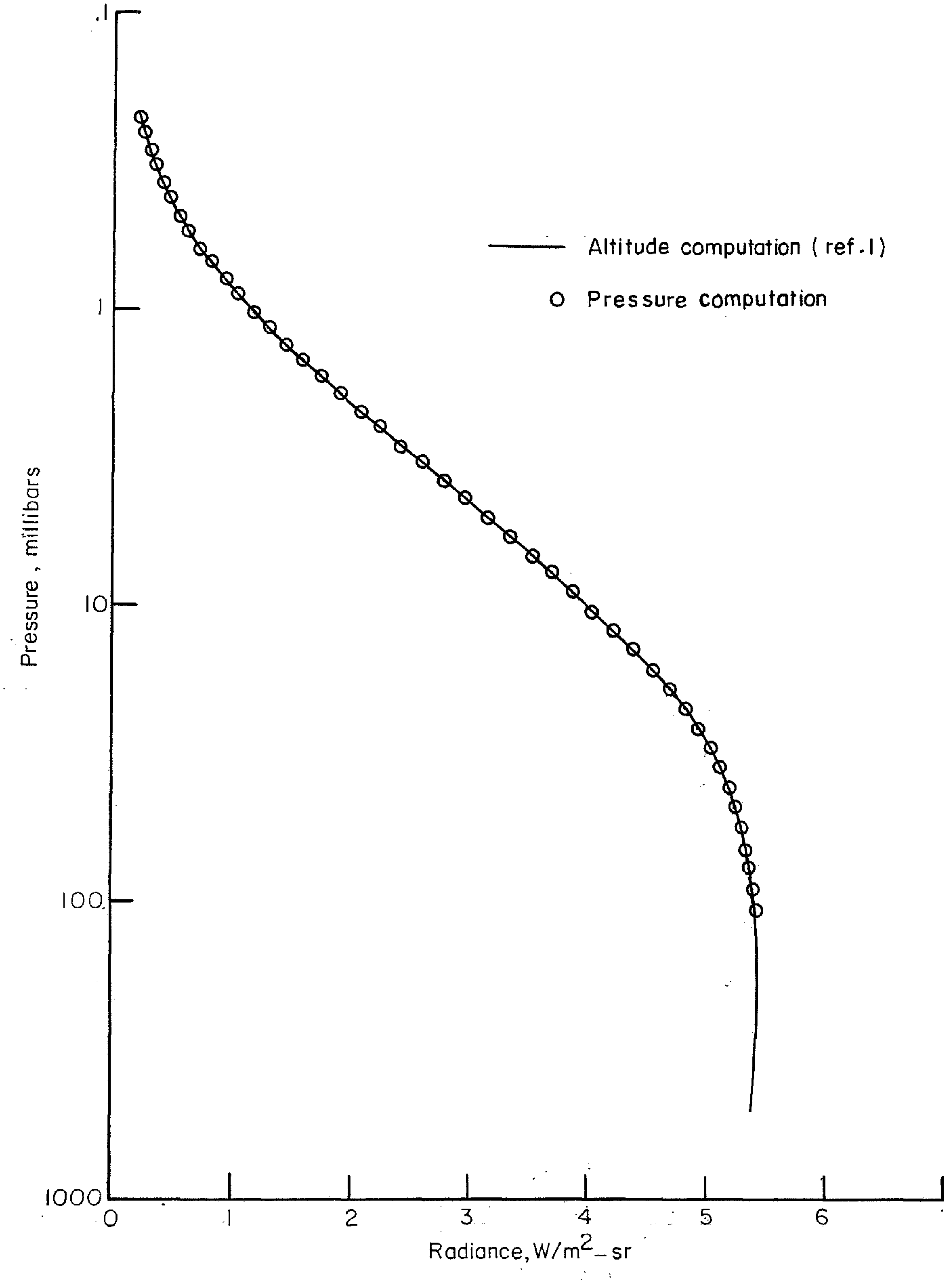


Figure 2.- Comparison of limb radiance profile computed by using altitude with that computed by using pressure for 615-cm^{-1} to 715-cm^{-1} spectral region.

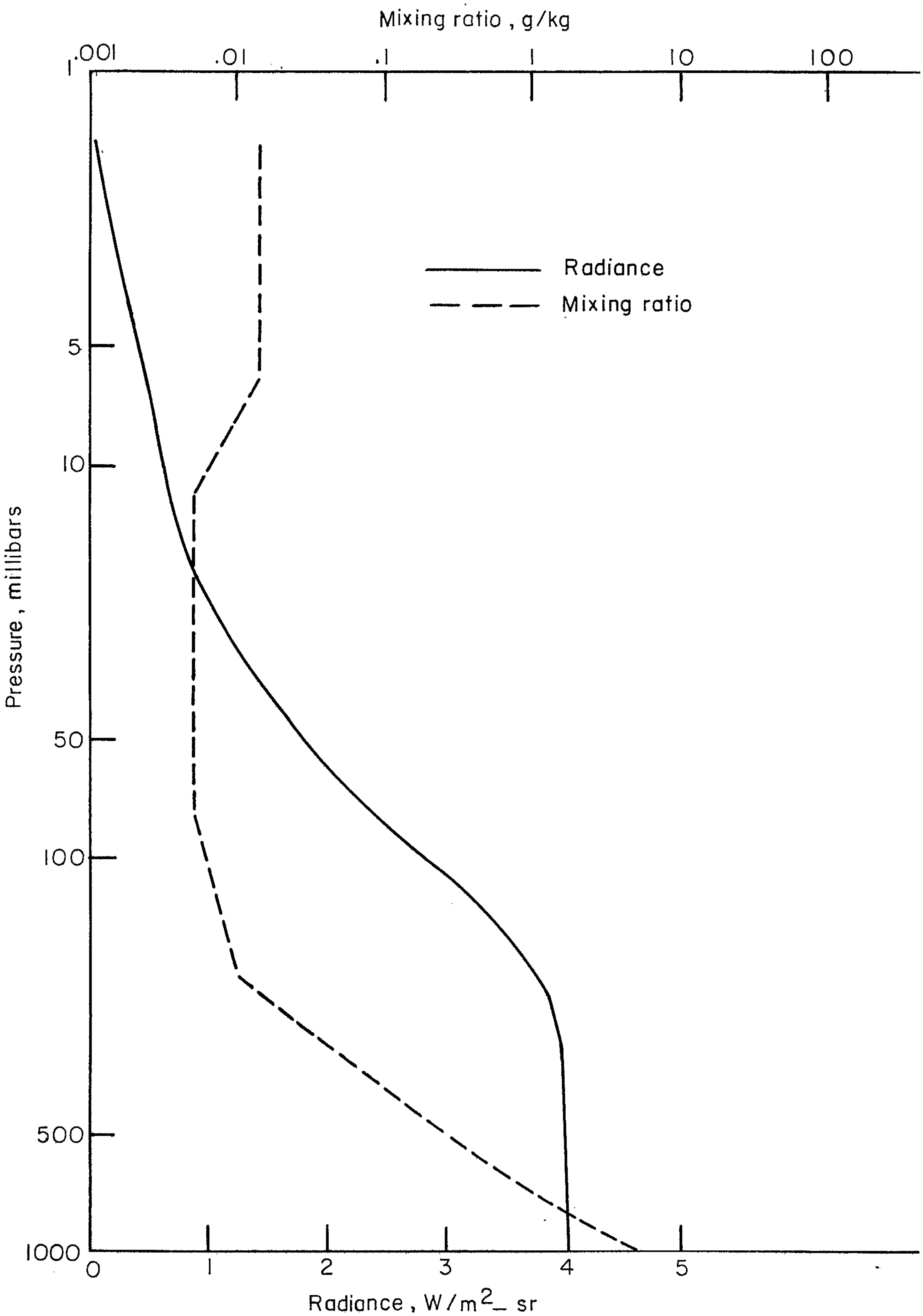
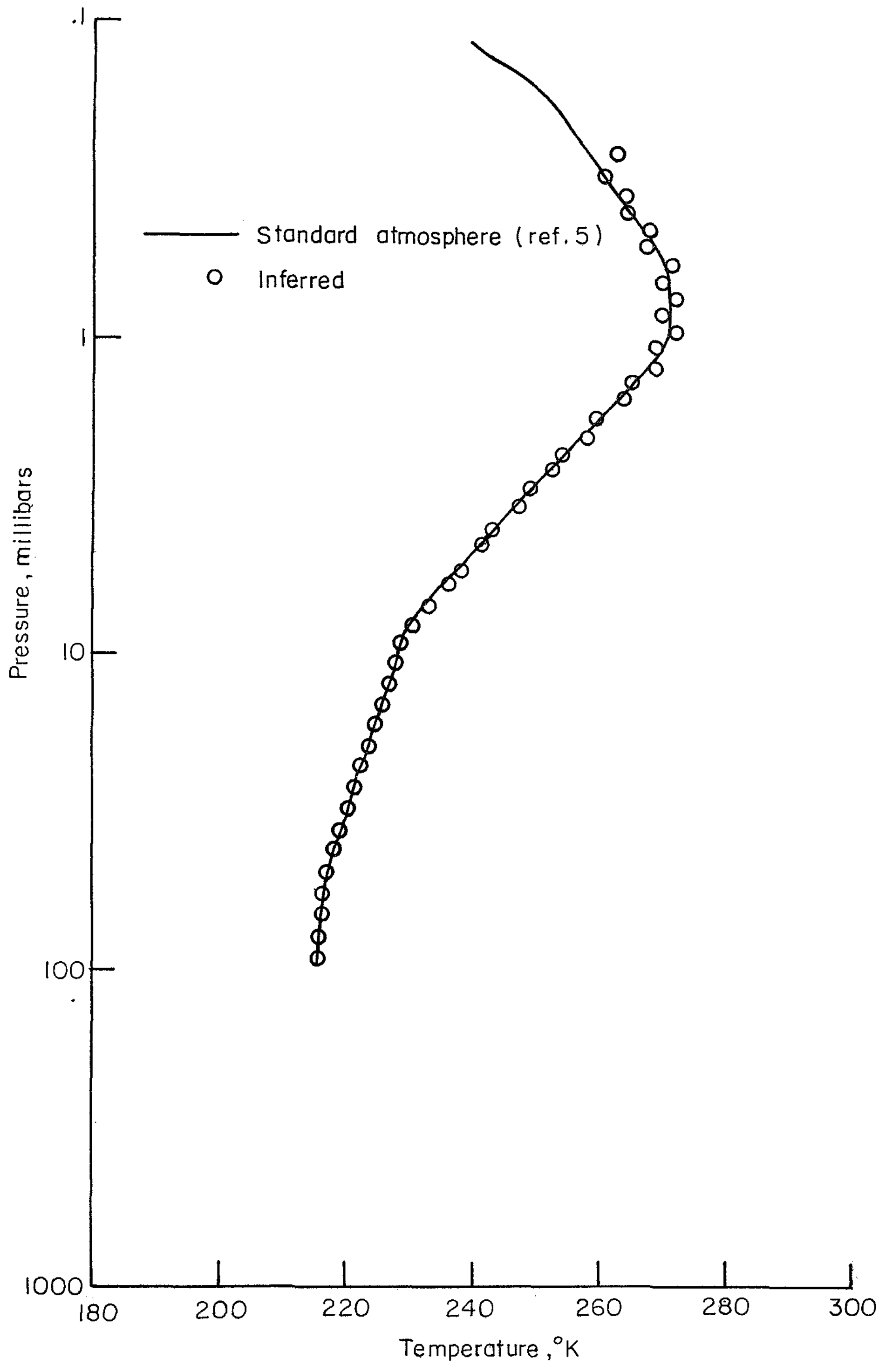
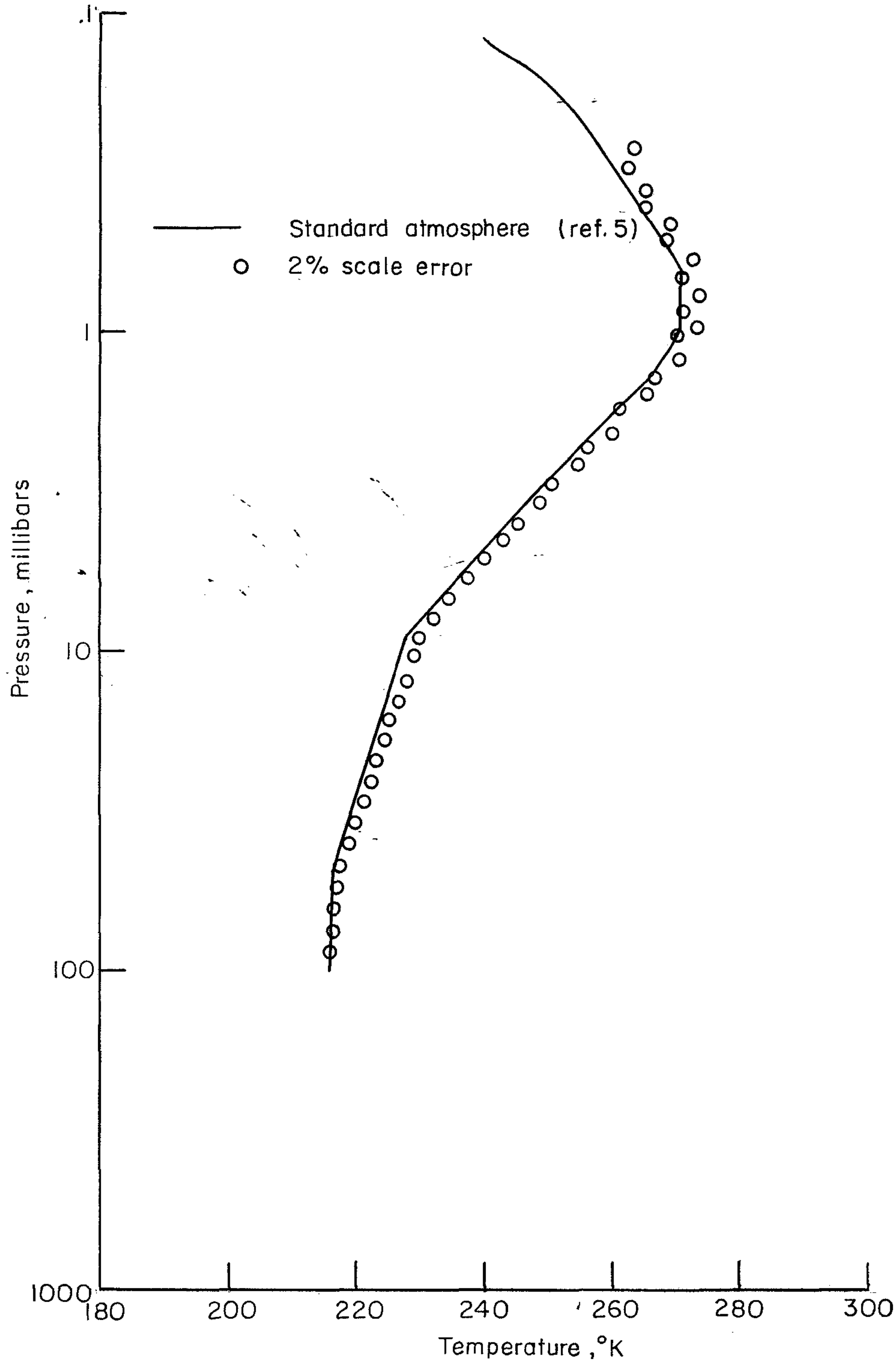


Figure 3.- Limb radiance profile and water-vapor mixing ratio for 205-cm^{-1} to 295-cm^{-1} spectral region.



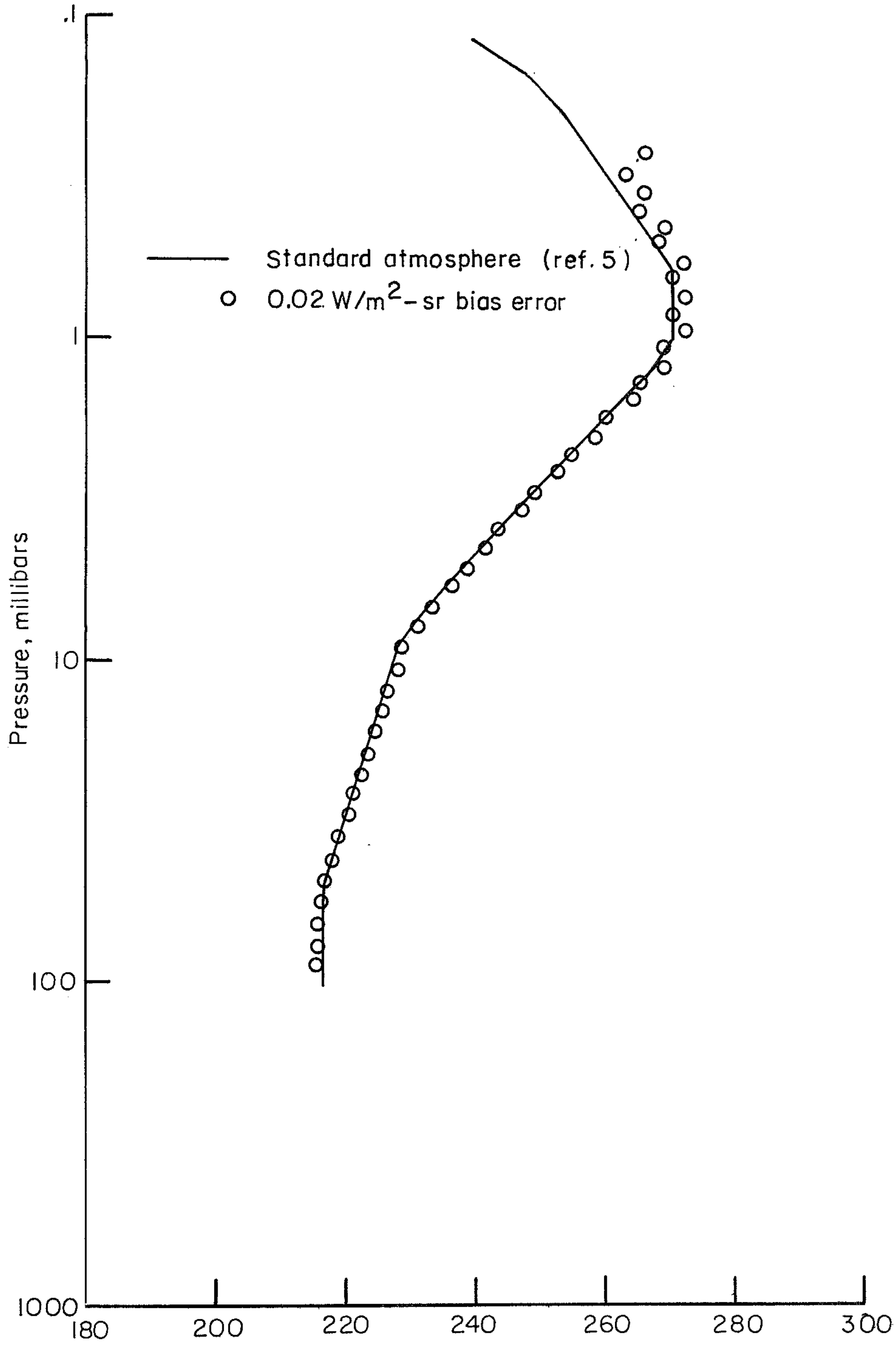
(a) No radiance error.

Figure 4.- Inferred temperature profiles.



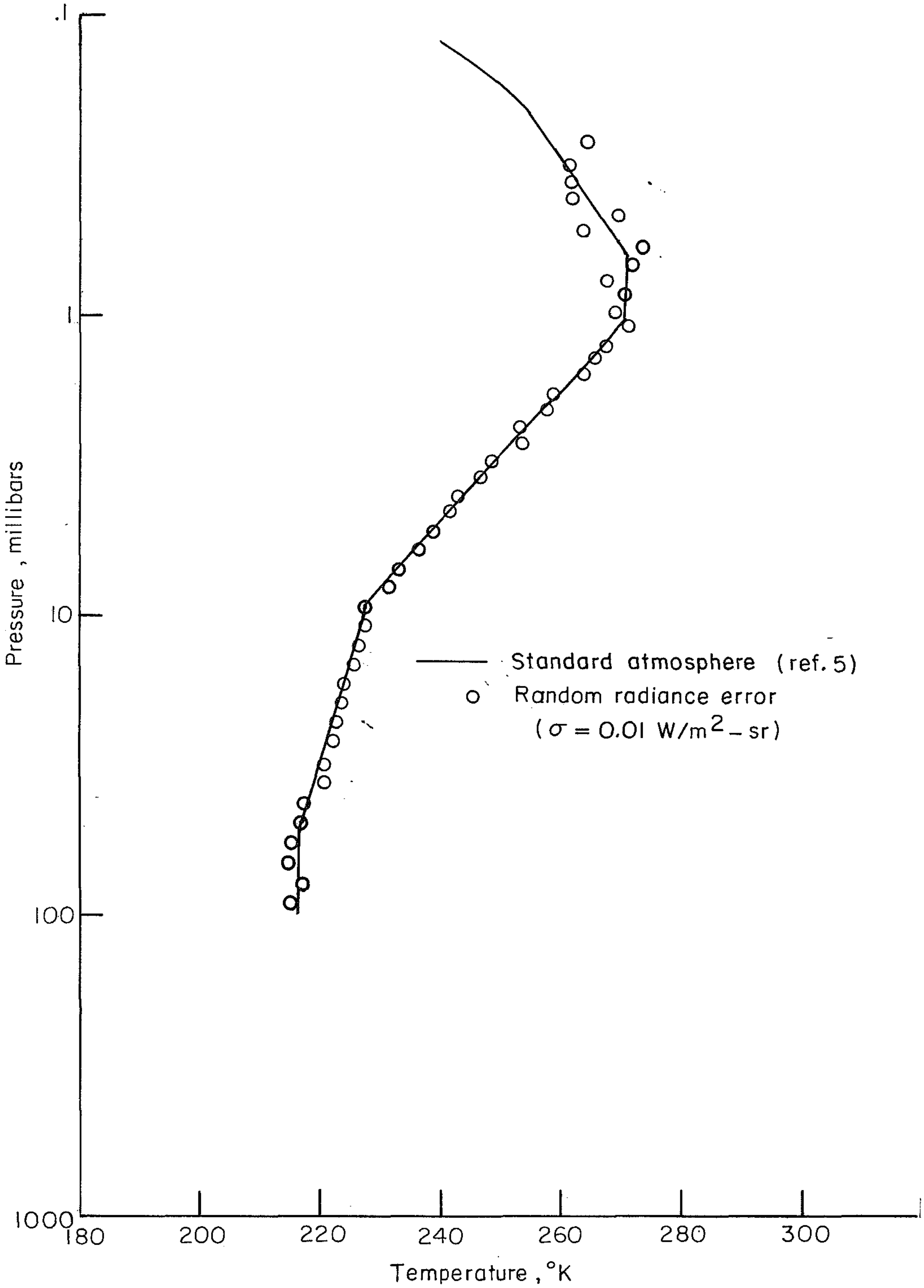
(b) Scale radiance error.

Figure 4.- Continued.



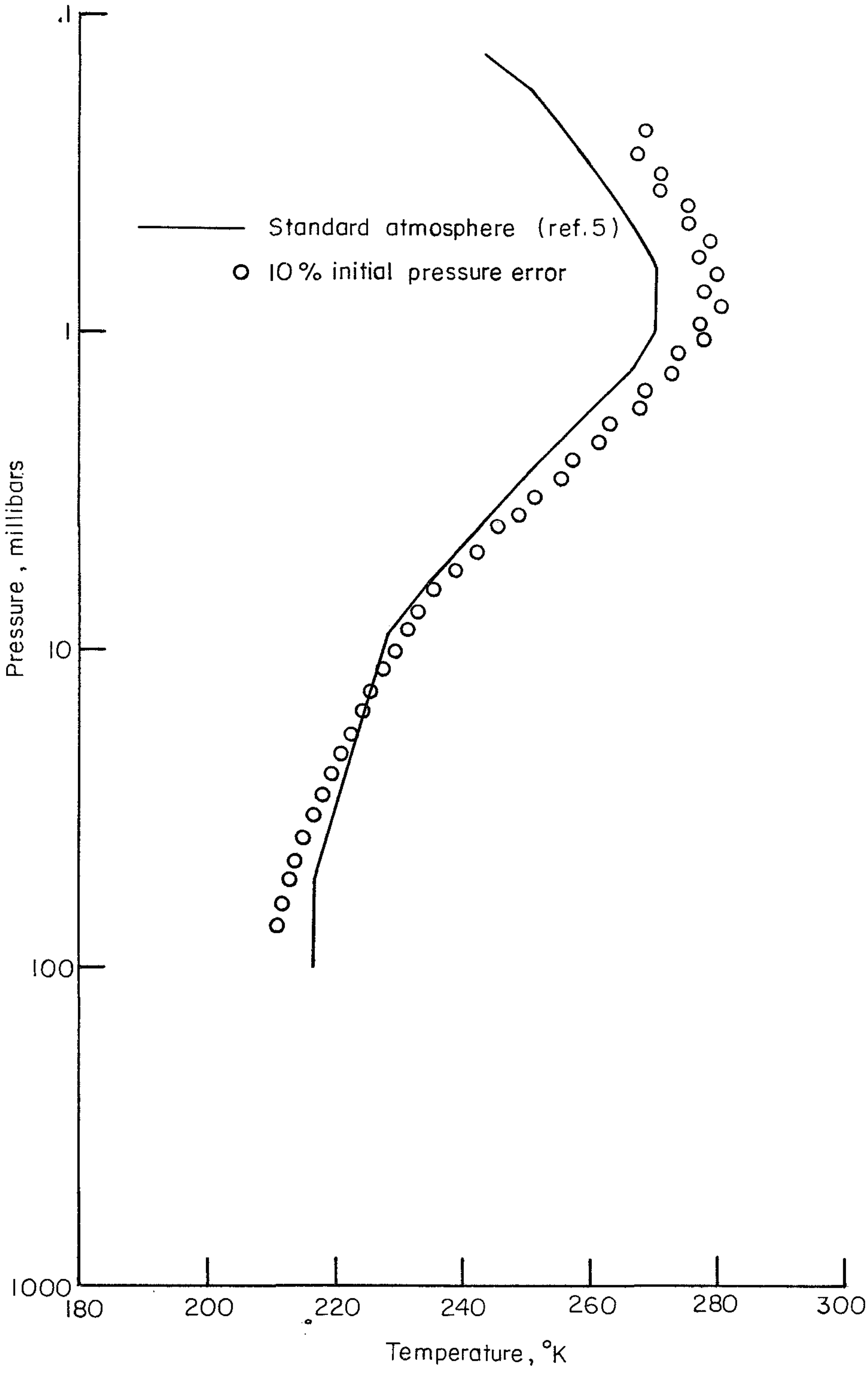
(c) Bias radiance error.

Figure 4.- Continued.



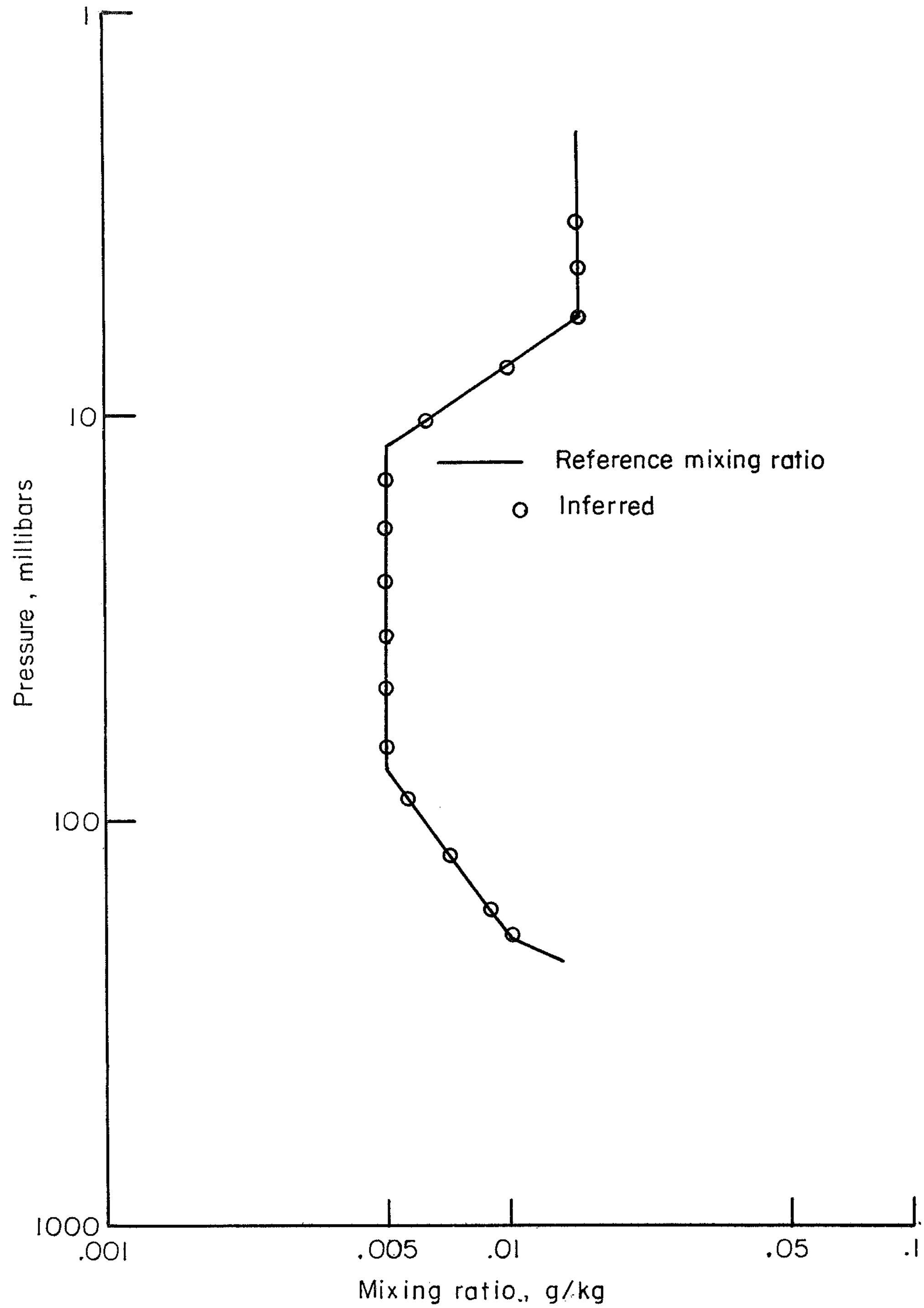
(d) Random radiance error.

Figure 4.- Continued.



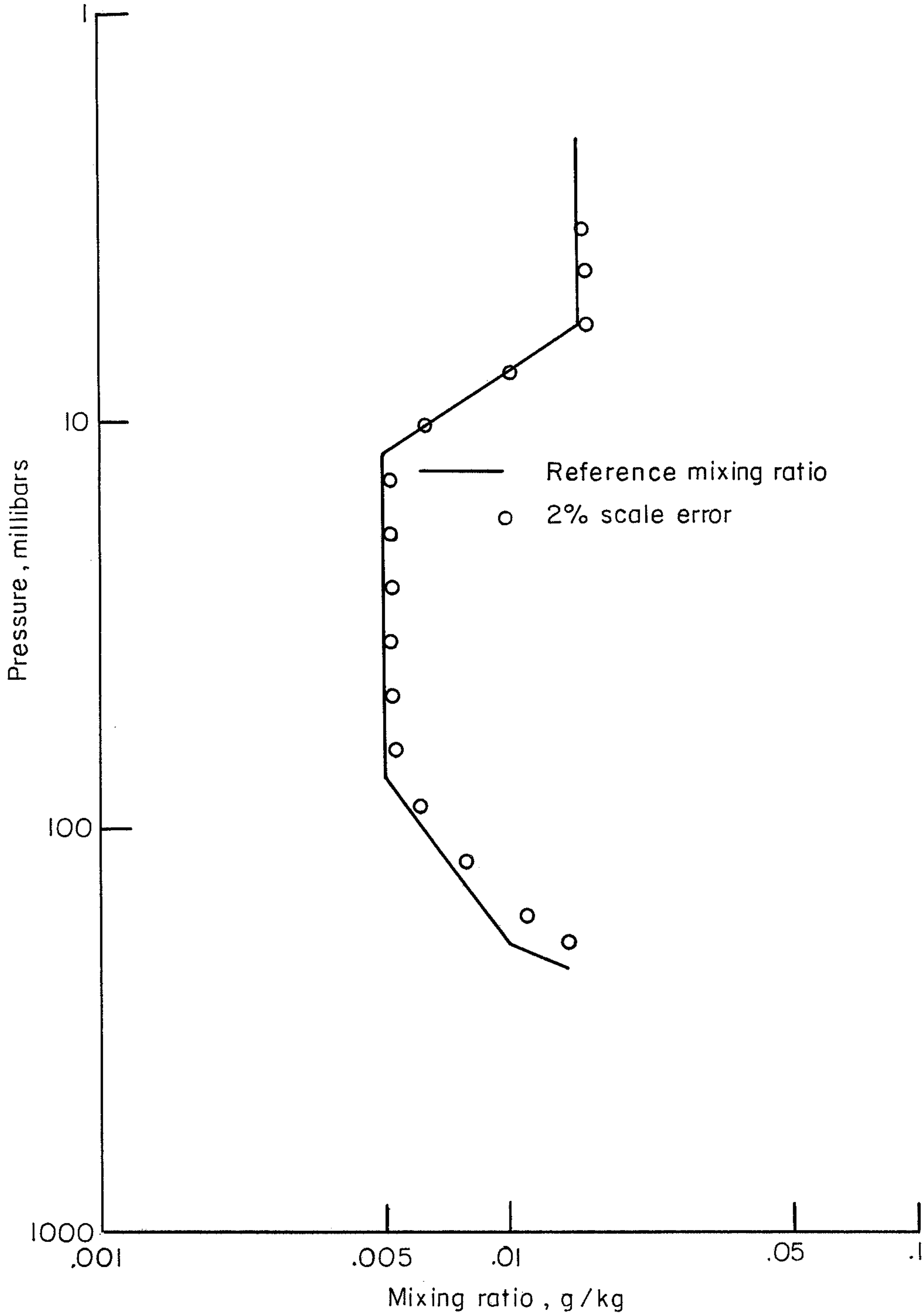
(e) Initial pressure error.

Figure 4.- Concluded.



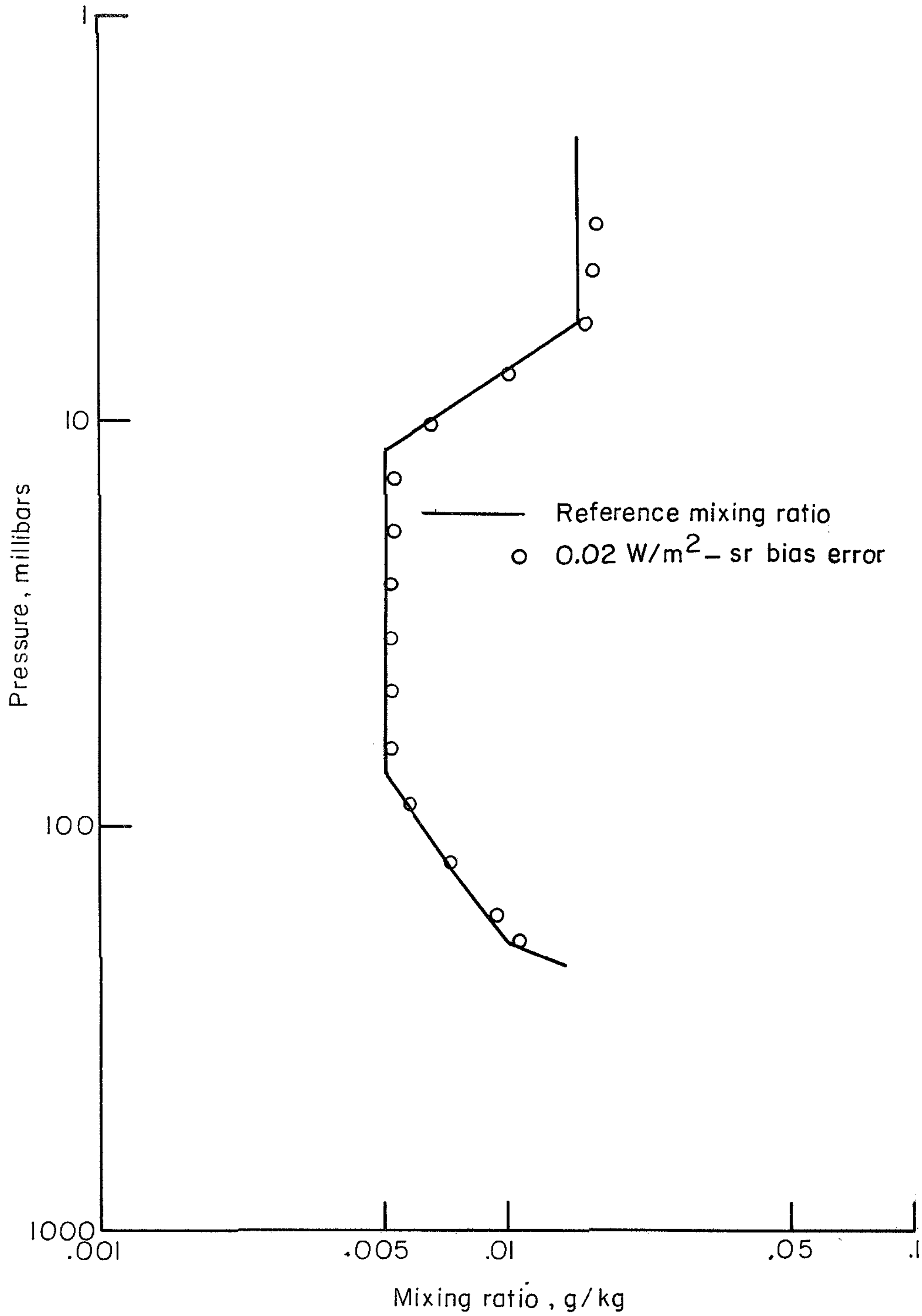
(a) No radiance error.

Figure 5.- Inferred water-vapor mixing ratio.



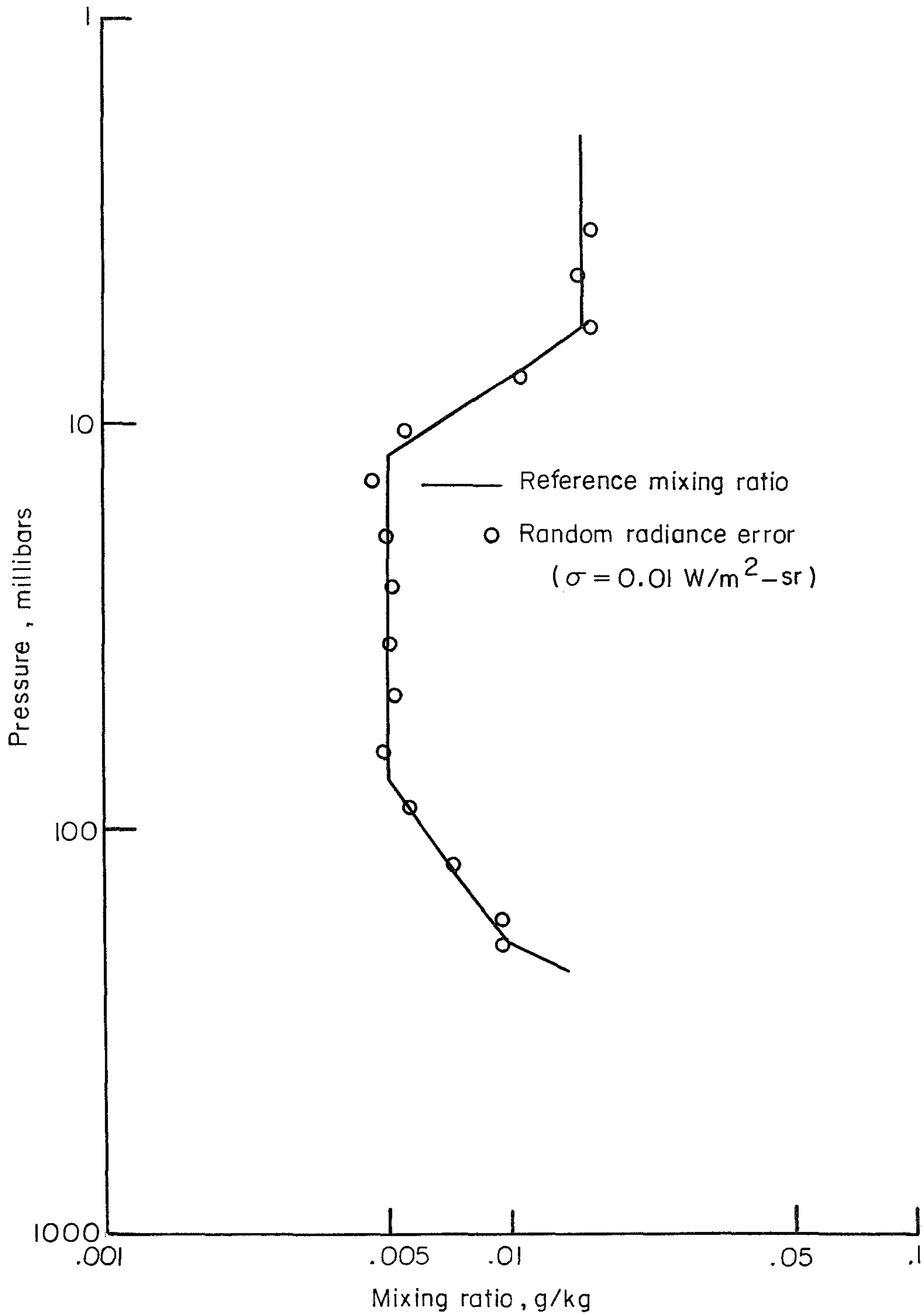
(b) Scale radiance error.

Figure 5.- Continued.



(c) Bias radiance error.

Figure 5.- Continued.



(d) Random radiance error.

Figure 5.- Concluded.

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